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TTCS Condenser Freezing Test Report

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Summary

For the AMS experiment onboard the International Space Station a thermal control system, known as the Tracker Thermal Control System (TTCS) is being developed. The TTCS basically consists of a mechanically pumped two-phase loop, where heat is collected at two evaporators and rejected at two radiators. The loop contains carbon dioxide (CO_2). The location of the main parts of the loop has been chosen in such a way that freezing of these parts is inherently impossible. However, during (accidental) total power down of the experiment, the condenser attached to the tracker radiators may freeze. In a worst case situation, where the condenser feed and return lines are still frozen and thus blocking the condenser in and outlet, pressure builds up during thawing of the condenser.

This document describes the results from tests carried out to determine the TTCS condenser maximum design pressure (MDP).

From the tests it is concluded that the pressure build up during thawing -while condenser in and outlets are blocked- follows the CO_2 melting line found in the CO_2 3-phase diagram. Therefore the condenser maximum design pressure directly follows from this diagram once the maximum non-powered condenser temperature is known. From thermal analysis calculations this temperature was found to be -5 °C leading to a condenser MDP of 3000 bar. A condenser consisting of small Inconel 718 tubing (d_{in} = 1.0mm, d_{out} = 3.0mm) is shown to withstand this pressure within applicable safety margins.

Document Change Log

Issue	Description	Page no.	Date	By
1.0	First issue	-		All authors
2.0	- Error in the conclusion corrected: the von Mises stress was accidentally taken from the 2000 bar line in the graph. Should be taken from the 3000 bar line. - Various typing errors.		July 2005	G. van Donk/J. van Es
3.0	- Summary lines added - Document change log added - Ref. 5 added - Deviations from test plan added - Figure 7-2 replaced and depending conclusions updated		August 2005	G. van Donk

(31 pages in total)

1 Introduction

1.1 General

During (accidental) AMS experiment power down the TTCS condenser and part of it's feed and return lines may freeze. If no appropriate measures are taken the condenser eventually may burst during thawing. To avoid this, a condenser is being designed comprising a number of capillary tubes in parallel, attached to an interface plate which in turn is bolted to the radiator heat pipes.

In order to perform the final sizing and material selection of the condenser, the maximum design pressure (MDP) inside the condenser during thawing is needed. It was decided to do a test to determine this pressure. The corresponding test plan can be found in document "AMSTR-NLR-TN-022 issue 3.0" [Ref. 4]. The document before you presents the results of the execution of this test plan.

1.2 Document structure

The objective of the AMS-02 condenser freezing test is stated in chapter 2. Chapter 3 provides a description of the test set-up. Details concerning the calibration of the pressure sensors are discussed in chapter 4. A step-by-step description of the tasks performed to successfully execute the test is provided in chapter 5. In chapter 6 the test results are presented and discussed. Finally, in chapter 7 conclusions are drawn.

2 Objective

The main objective of the tests is to find a method to determine the condenser maximum design pressure (MDP). This pressure occurs during heat-up and thawing of the condenser after being cooled down, while the feed lines still remain frozen.

A second, and related, objective is to get a better general understanding of CO_2 freezing and thawing phenomena.

3 Test approach and test set-up

3.1 Test approach

In the condenser, maximum pressure build-up will occur during thawing while the feed and return lines are still frozen. It is expected that this pressure will be very high and to keep the condenser mass as low as possible, using small diameter tubing as part of the condenser design is believed to be an effective way (if not, the only way) to withstand this pressure. CO_2 has a free expansion of 28 vol% when changing from the solid to the liquid state [Ref 6] . The expansion of a metal tube is negligible wrt this value, regardless what metal is being used and therefore pressure will rise. To determine this pressure a **cold forged stainless steel 316Ti tube (d_{in}=1.0mm and d_{out}=3.0mm) with relatively high strength**, equipped with 8 strain gauges was used. This tube remains elastic up to 2500 bar, limiting the test to this value. Prior to testing, the strain gauges were calibrated by applying internal fluid pressure on the tube. After calibration, the tube section with the straingauges can be considered a pressure sensor.

3.2 Set-up Description

A downscaled CO_2-loop was built and tested in an environment well below the freezing point of CO_2. Figure 3-1 shows a schematic of the loop. The loop is equipped with a temperature controlled accumulator and a capillary stainless steel 316Ti tube, partly equipped with strain gauges. This part will be referred to as the 'measurement section' and can be temperature controlled independently from the other components. The number of couplings, branches and intrusive test components are minimised to avoid accidental leak. The loop was evacuated and then filled with CO_2. While being pressurised, by controlling the accumulator temperature, the 'measurement section' was cooled below the freezing point of CO_2. Cooling was established by feeding N_2 gas into the aluminium box that contains the measurement section. The feed and return lines were cooled below the CO_2 freezing point as well, starting from the condenser side. For this purpose the feed & return lines are equipped with zone heaters which were switched off one after another to create a 'propagating freezing front'. After a certain dwell time the measurement section was temperature controlled to a temperature above the freezing point, while watching and recording the strain gauge outputs.

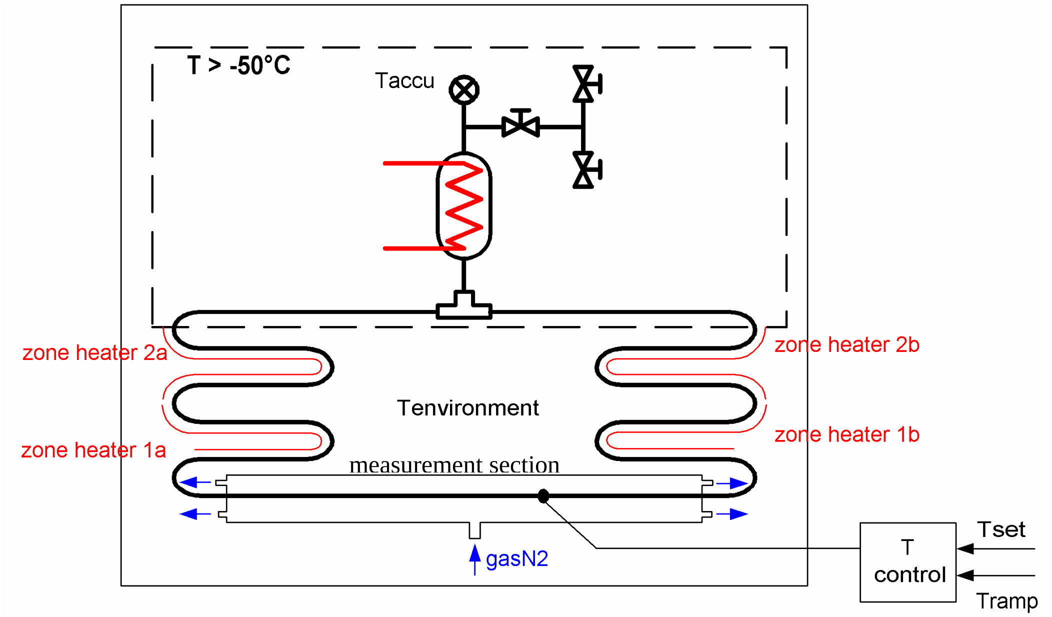


Figure 3-1: Test set-up schematic

A number of measurement thermo-couples were applied over the length of the CO_2 tubing. Additionally, seven control thermo-couples were used to provide direct feedback to the temperature control equipment. The location of these couples are shown in Figure 3-2. The zone heaters are indicated in Figure 3-1 and Figure 3-3.

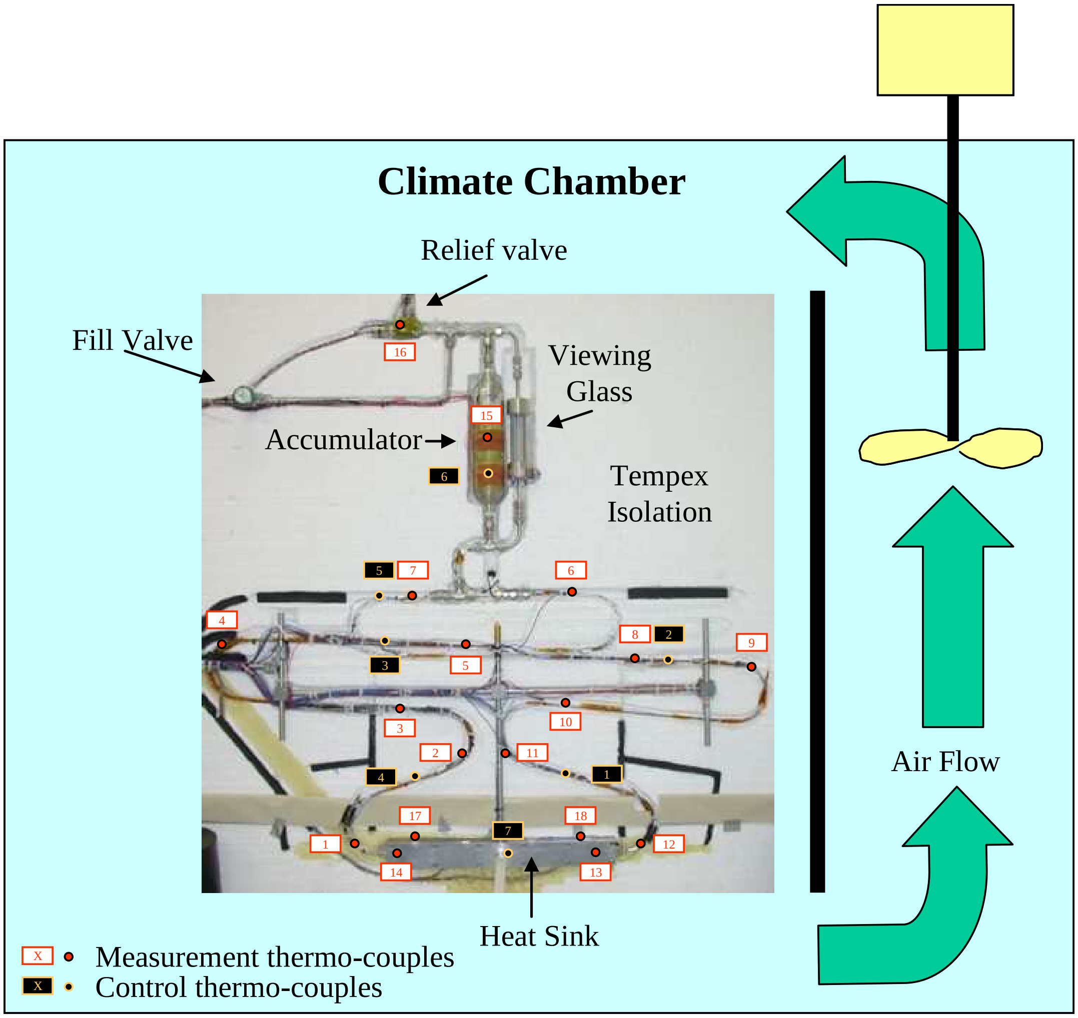


Figure 3-2: Test set-up, locations of thermo couples

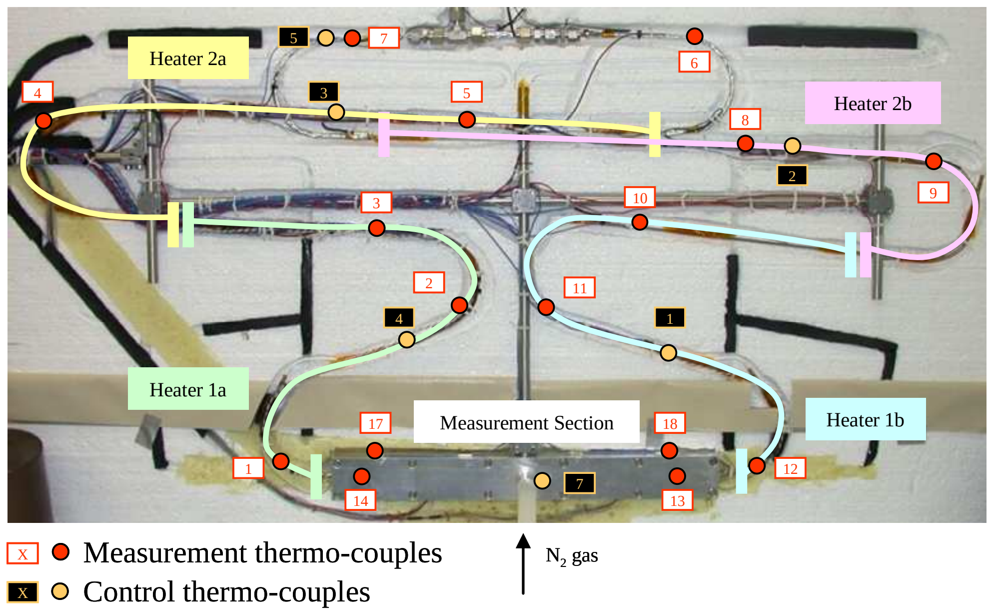


Figure 3-3: Test set-up, locations and sizes of zone heaters

The tube inside the measurement section is equipped with 8 strain gauge bridges, see Figure 3-4 and Figure 3-5. Each bridge consists of two EA-06-050TG-350 strain gauges with measuring grids in two orthogonal directions. The grids are connected in a Wheatstone bridge: two grids in the tangential direction and two grids in the longitudinal direction of the tube. The bridges are covered with a protective coating and are compensated for zero-shift with temperature. Due to the strong curvature of the tube the bridges have large initial zero balances and the remaining zero shift with temperature is very non-linear. With an excitation of 5 Volt and the given dimensions and material of the tube the theoretical sensitivity of the bridges is 0.433 mV/1000 Bar. The actual sensitivities have been calibrated at three different temperature levels: -18.5, -54.5 and -68.7°C as will be discussed in paragraph 4.2. Also, the actual zero shifts with temperature have been determined for the temperature range -120°C ↔ 30°C. This will be discussed in paragraph 4.1.

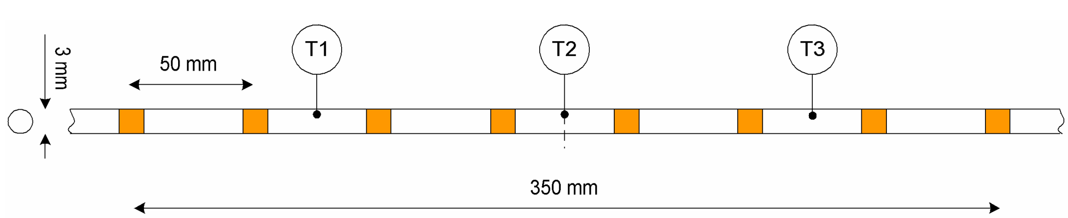
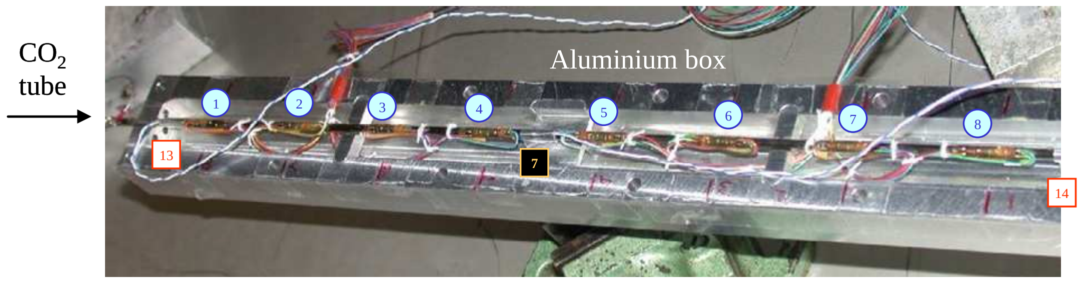


Figure 3-4: Measurement section (here drawn without aluminium box)



X Measurement thermo-couples

7 Control thermo-couple

X Strain Gauges

Figure 3-5: Measurement section, locations of strain gauges

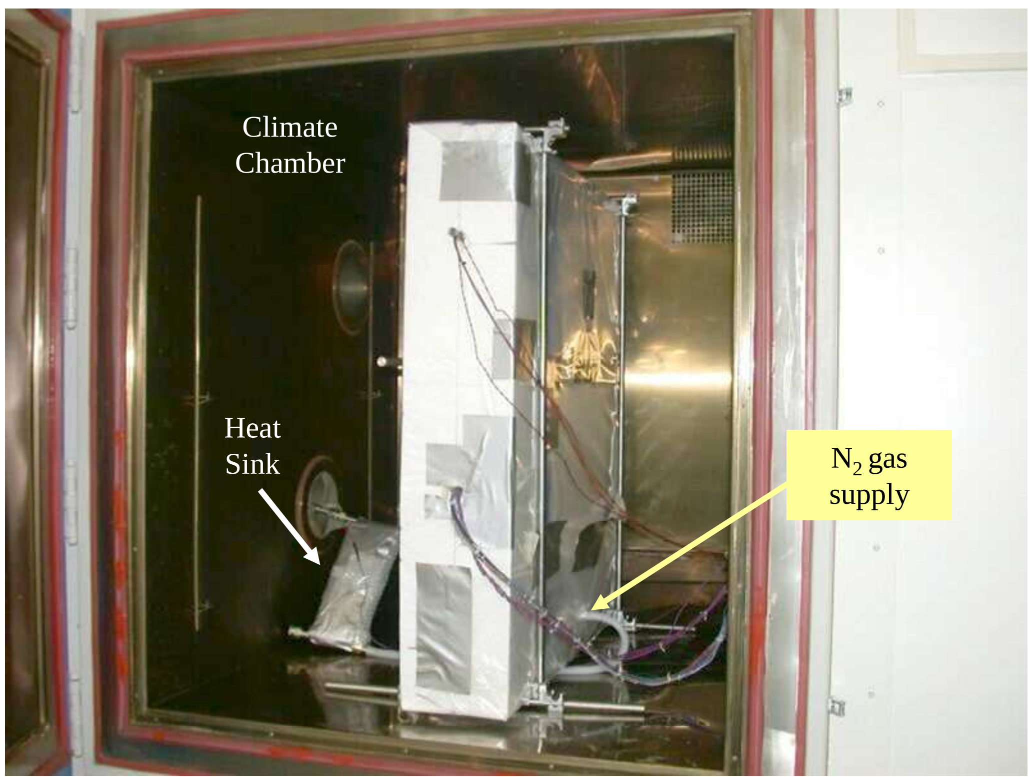


Figure 3-6: Test set-up integrated in climate chamber

3.3 Equipment list

The table below provides a list of equipment used to carry out the test. Figure 3-7 shows some of the equipment.

Table 3-1: CO_2 freezing/thawing test equipment

Equipment description	Parameter	NLR ID	SER #	Calibration or check date	Date due
Climate chamber CH1200 LN2	Environment temperature	13.09	7336	25032005	25032006
Thermocouple voltmeter, Keithley 2001	Temperatures	3.42		15022005	15022006
TC ref. Junction, Dostman, incl Pt100	Temperature	9.39	862A	26102004	26102004
8x Strain gauge conditioning units MK3	Strain	9.101 to 9.108	-	Feb 2005	Feb 2006
Power supply E030-10	Accu heating	06.50		-	-
Power supply E030-10	Tube T-split heating	06.51		-	-
Power supply E030-10	Tube heating	06.24		-	-
Power supply E030-10	Tube cooling (valve power)	06.48		-	-
Power supply SM 7020D	Relief valve heating	06.08		-	-
Power supply SM 3540D	Zone heating			-	-
4x Eurotherm 2416	Zone temp control			-	-
2x Eurotherm 2416	Accu temp & Tube T-split temp control	09.29		-	-
Eurotherm 2216e	Condenser Tube temp control	09.31		-	-
Thermocouple type T	Temperature	n.a.		Premium grade	

Figure 3-7: Test equipment

4 Calibration of Strain Gauges

Measures were taken to inherently compensate the strain gauges for temperature effects. Still, some temperature sensitivity remained. A test has been carried out to calibrate the gauges' output over the envisaged temperature range that will be applied to determine the maximum design pressure of the condenser.

For the calibration test a hand pump with analog read-out was used to apply pressure to the test set-up and the output voltages of the strain gauges were measured at three different temperatures. Figure 4-1 shows a proof-of-concept set-up for this test.

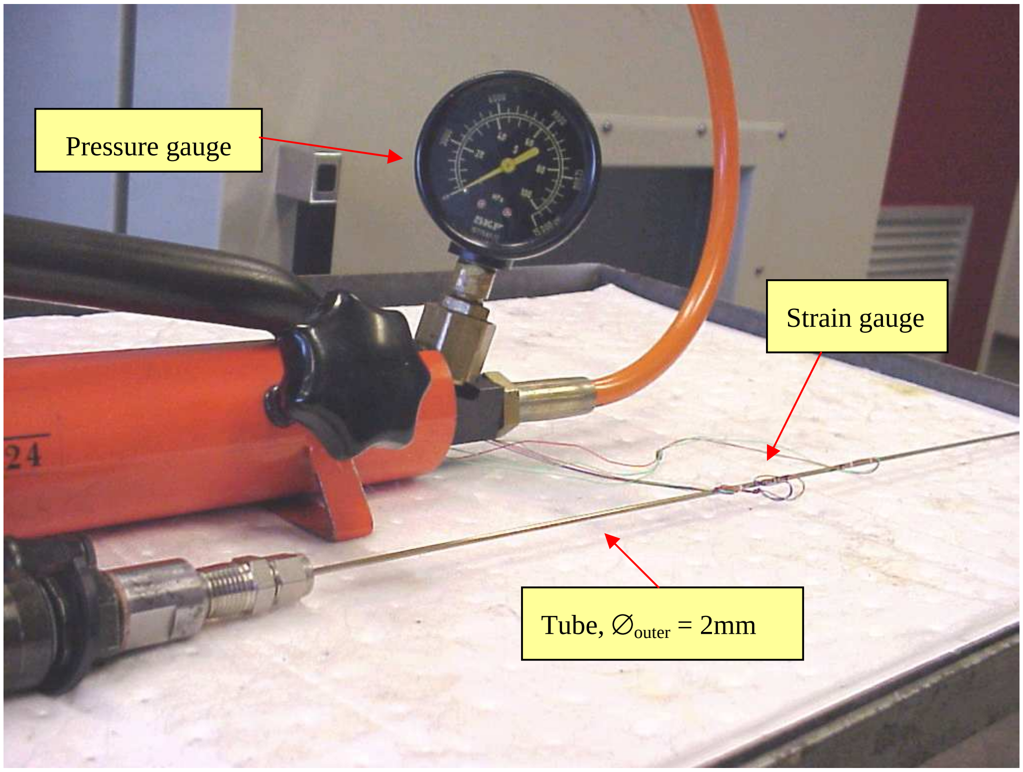


Figure 4-1: Pre-test setup for Strain gauges calibration (here shown with different tube diameter than actually used)

The calibration test was performed in two parts; an unpressurized and a pressurized measurement of the strain gauges. In the subsequent paragraphs each part of the test is discussed.

4.1 Strain gauge temperature sensitivity check (unpressurized)

In the first part the test set-up was empty (unpressurized) and in a controlled fashion cooled down to a minimum temperature of -120°C. After a short interval -to allow the temperature of the various sections to settle- the temperature was raised back up to ambient. During this process the voltage outputs of the strain gauges were continuously measured. Figure 4-2 shows for each strain gauge the measured voltage plotted against temperature. Second order polynomial curves have been fitted through the data. The polynomial coefficients are listed in Table 4-1.

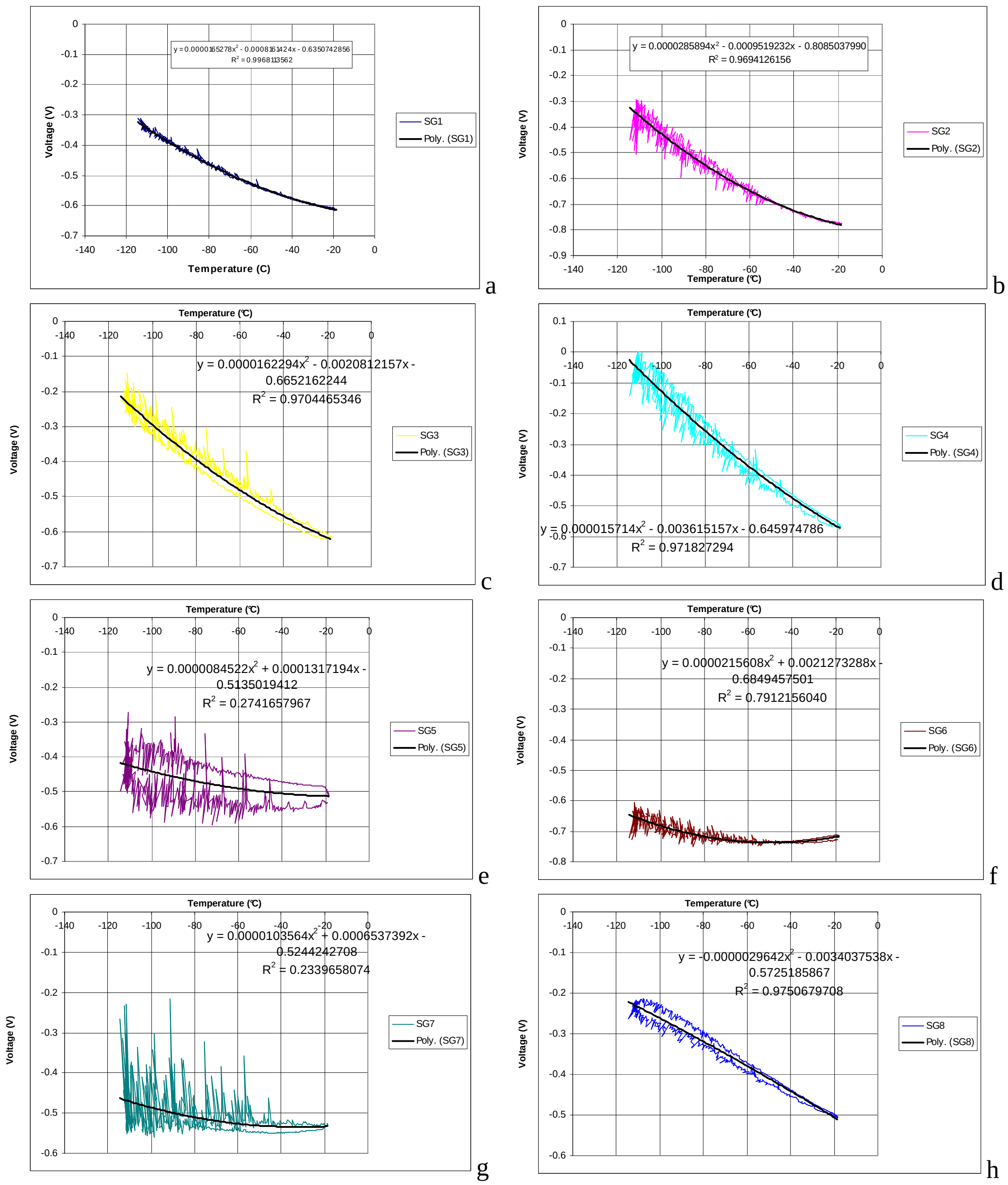


Figure 4-2a-h: Strain gauge temperature calibration curves (unpressurized)

Table 4-1: 2nd order calibration curve-fit polynomial coefficients

	SG1	SG2	SG3	SG4	SG5	SG6	SG7	SG8
X^0	-0.6350742856	-0.808503799	-0.665216224	-0.645974786	-0.513501941	-0.68494575	-0.524424708	-0.572518587
X^1	-0.0008161424	-0.000951923	-0.002081216	-0.003615157	0.000131719	0.002127329	0.000653739	-0.003403754
X^2	0.0000165278	2.85894E-05	1.62294E-05	0.000015714	8.4522E-06	2.15608E-05	1.03564E-05	-2.9642E-06

4.2 Strain gauge pressure calibration at 3 temperatures

The output voltage of all eight strain gauges were measured while applying pressures of subsequently 0, 200, 400, 600, 800, 1000, 800, 600, 400, 200 and down to 0 again. This was repeated at three different temperatures. The maximum applied pressure (1000 bar) was limited by the handpump, the tube remains elastic up to 2500 bar. The measurement results are plotted in Figure 4-3, Figure 4-4 and Figure 4-5. Table 4-2 summarizes the strain gauge voltage-to-pressure conversion factors for each of the temperatures.

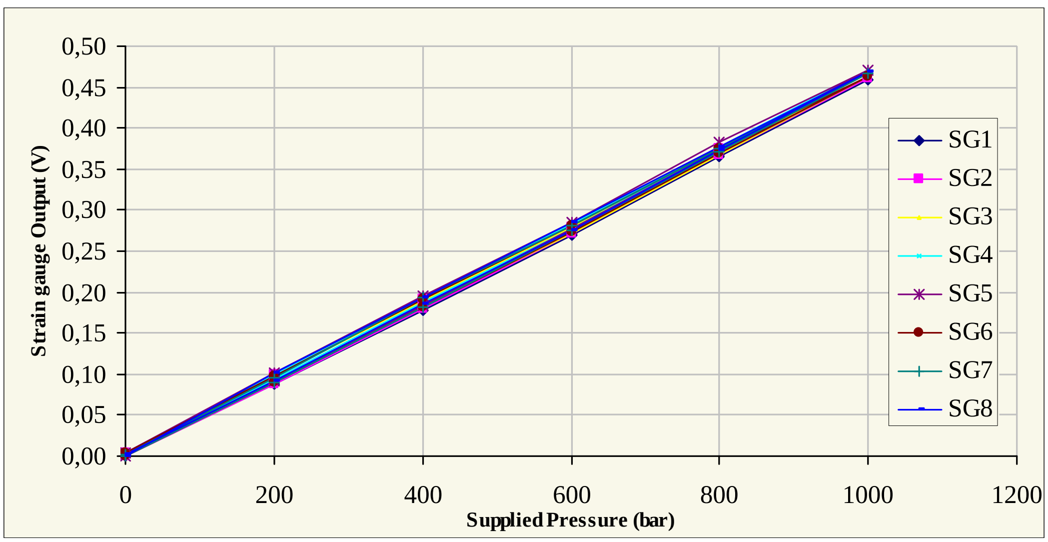


Figure 4-3: Strain gauge pressure calibration @ T=-18°C

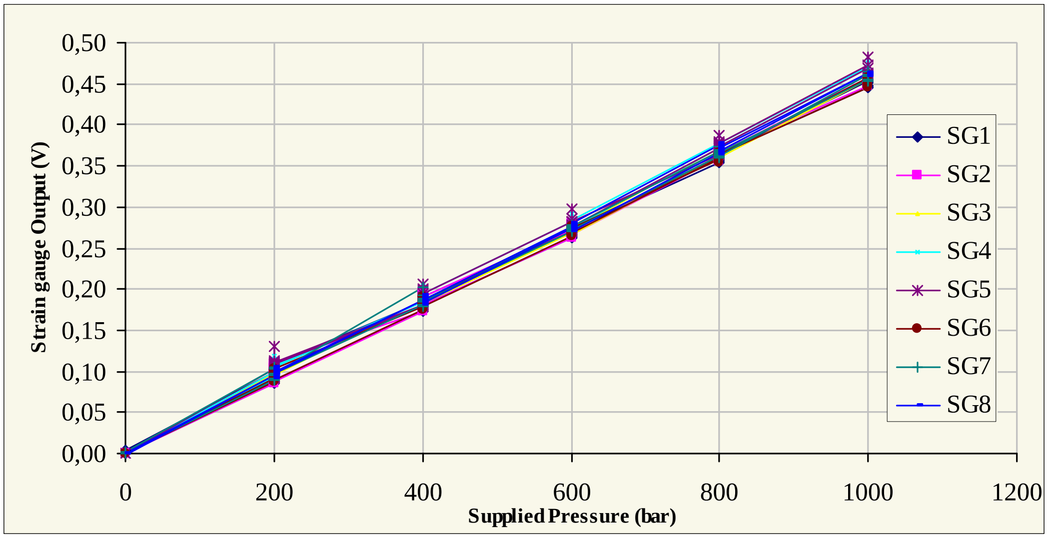


Figure 4-4: Strain gauge pressure calibration @ T=-54°C

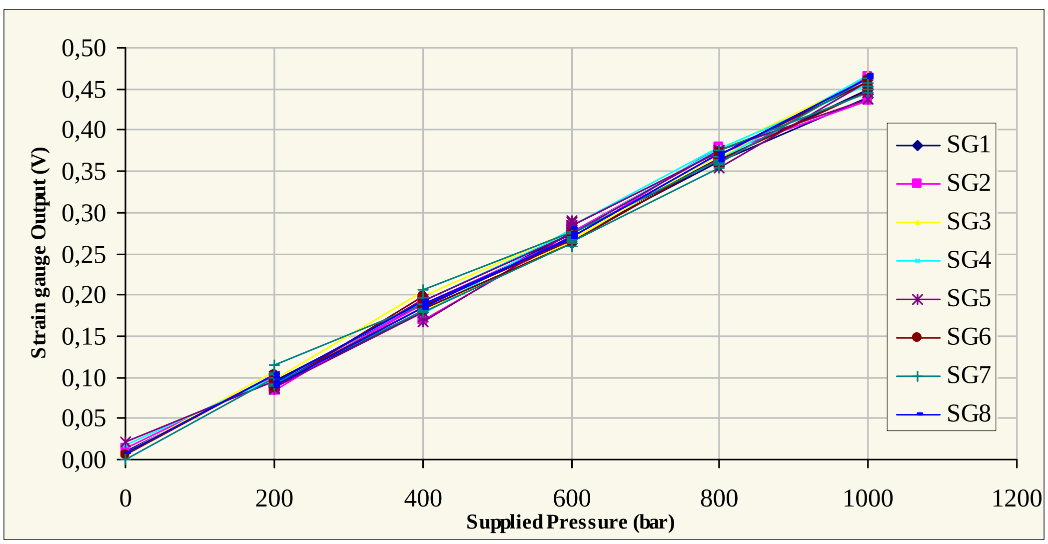


Figure 4-5: Strain gauge pressure calibration @ T=-68°C

Table 4-2: Strain gauge Voltage to Pressure conversion factors

	Temp (°C)	SG1	SG2	SG3	SG4	SG5	SG6	SG7	SG8	
	-18,56	2189	2177	2171	2139	2135	2168	2147	2146	bar/V
	-54,46	2266	2240	2222	2182	2203	2235	2218	2196	bar/V
	-68,73	2246	2192	2186	2173	2213	2205	2250	2174	bar/V
Average	**-47,25**	2233	2203	2193	2164	2183	2202	2204	2171	bar/V

Because of the fact that the pressure had to be kept constant by hand and the pressure gauge readout had to be done manually (as opposed to electronically), some readings show quite a large variation. However, every data point was gathered multiple times, so random variations level out when averaged. As expected, the strain gauge output is linear with pressure and extrapolation is justified as long as the tube is elastic, e.g. up to 2500 bar.

5 Test Execution

5.1 Deviations from the test plan

For several reasons the actual test execution deviates somewhat from the test plan “AMSTR-NLR-TN-022-issue02” [Ref.4]. Differences are listed here below.

#	Deviation from test plan	Reason why
1	At the end of the test, the measurement section was not increased to +25 °C	- we did not want to destroy the test setup after all. (Possible plastic deformation of the test tube). - it appeared not to be possible to heat up the measurement section, without heating up the feed and return lines
2	Thawing sequence of feed & return line zones sometimes reversed	- temperature distribution along feed & return lines appeared not to be linear but quite random due to relative hot N2-gas leakage from measurement section. A nice propagating ‘thawing front’ therefore was not possible and the original zone heater switch on sequence seemed not so important
3	Generally shorter dwell times were applied	- during testing it became clear that the pressure followed the CO_2 melt line and therefore we were convinced that the CO_2 freezing phenomenon was understood. Besides, other dwell times are rather arbitrary and there is no reason why this would

		change the results.
4	Some details, for example climate chamber temperatures (see revised procedure)	- to speed up the test, without compromising. During testing, depending on the results, generally it becomes clear when to (slightly) alter the envisaged test plan

5.2 Updated test procedure

Table 5-1 lists the updated procedure.

Table 5-1: Test procedure

Step	Action
1.	Perform He-leak test on test item prior to filling.
2.	Fill the system with CO_2
3.	Install the system into the climate chamber, including temperature sensors and power wires etc.
4.	Start data-acquisition system
5.	Set climate chamber and measurement section to 0 °C and accumulator to +5 °C
6.	Set climate chamber to –50 °C @ 5 °C/min
7.	Switch all zone heaters controls on to keep feed & return lines between –50 and –30 °C
8.	Wait for zone heater 1 temperature to become -30 °C
9.	Set measurement section to –120 °C @ 1.7 °C/min
10.	As soon as measurement section = -50 °C: Set climate chamber to –70 °C @ 0.4 °C/min
11.	After 30 minutes switch off zone heater 1
12.	After 30 minutes switch off zone heater 2
13.	After measurement section reaches –120°C wait until feed & return lines are –65 ° or lower
14.	Wait at least 1 hour
15.	Increase measurement section temperature with a maximum rate-of-change of 1.7 °C/min as long as tube section is elastic, simultaneously watch strain gauge outputs.
16.	If this is the third cycle AND the strain gauge outputs reproduced, further increase the measurement section temperature to +25 °C or until the tube bursts, whichever comes first.
17.	If this is the third cycle go to step 23
18.	Switch on zone heater 2, control zone temperature between –50 and –30 °C

Step	Action
19.	After 30 minutes switch on zone heater 1, control zone temperature between –50 and –30 °C
20.	After 30 minutes set climate chamber to –50 °C @ 5 °C/min
21.	Wait at least 1.5 hours
22.	Repeat from step 9, twice
23.	Set climate chamber to 10 °C @ 5 °C/min Set accu to +15 °C Set measurement section to +10 °C @ 5 °C/min
24.	Check CO_2 content in viewing glass
25.	Set climate chamber to 20 °C @ 5 °C/min Set accu to +20 °C Set measurement section to +20 °C @ 5 °C/min
26.	Remove test set-up and empty it
27.	Visual inspection

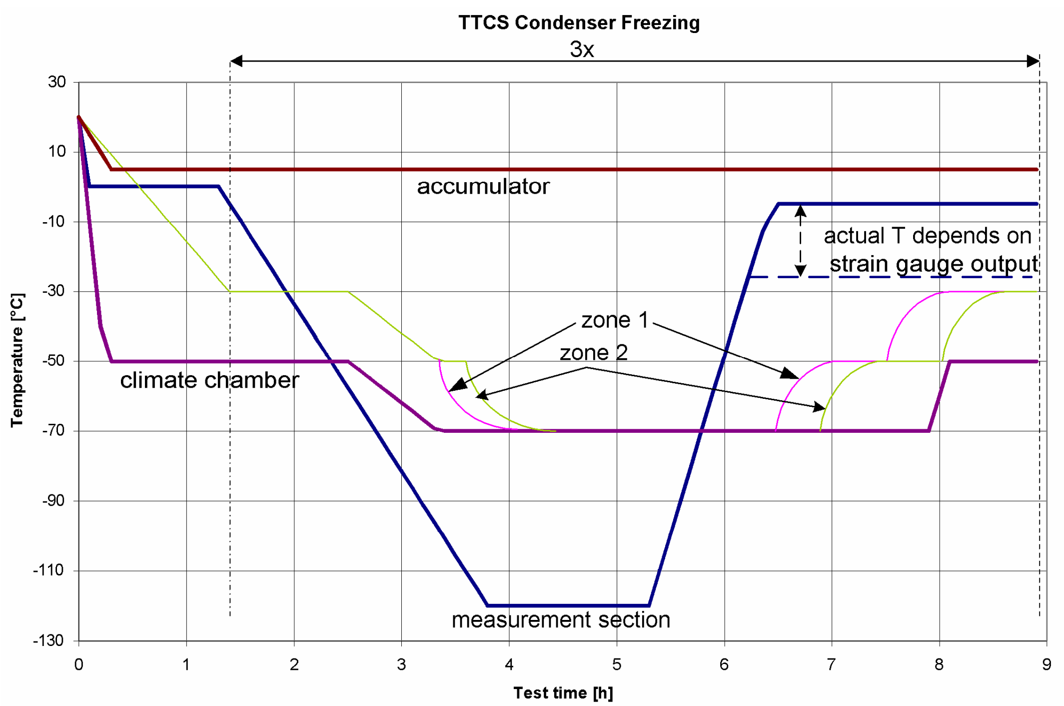


Figure 5-1: Freezing and thawing temperature cycle during a single test run

6 Test Results

6.1 General measurement explanation

In Figure 6-1 a typical freezing and thawing cycle of the test is indicated in the CO_2 P-T diagram shown. At starting point A the ambient pressure in the test set-up is set by controlling the temperature of the accumulator. Going from A to B the condenser is cooled to a temperature whereby CO_2 freezes, while the pressure is kept constant. After a certain time at point B the set-up has settled in temperature. Condenser and feed lines are now frozen. Next, the condenser is warmed up again. At point C the CO_2 in the condenser has reached the melting temperature and tries to change into liquid and expand. However, the feed lines are still frozen, trapping the melting CO_2. While temperature still increases, pressure starts to build up. When the feed lines finally thaw at point D, there is a sudden pressure drop as the overpressurized mixture of solid and liquid CO_2 can now push away the liquid in the feed lines and take up more volume.

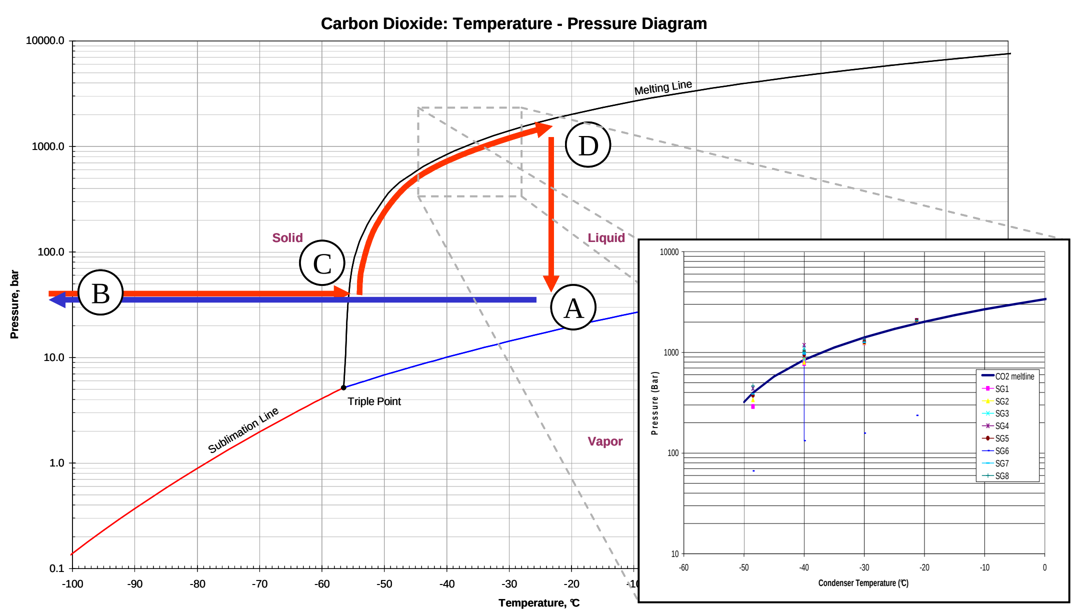


Figure 6-1: CO_2 melt line with measurement results

Note: the line D-A is an estimation. The actual process may possibly behave more along a line curved from D towards the melt line before returning to A.

The inset in Figure 6-1 plots the maximum pressures measured with each strain gauge in the condenser against the CO_2 melt line. See also Figure 6-7 for an enlarged version. It shows that the pressure build-up in the capillary tube follows the melt line. In this particular plot it is also clear to see that the output of strain gauge 6 is wrong. Most likely, the gauge was broken.

6.2 Typical Test Results

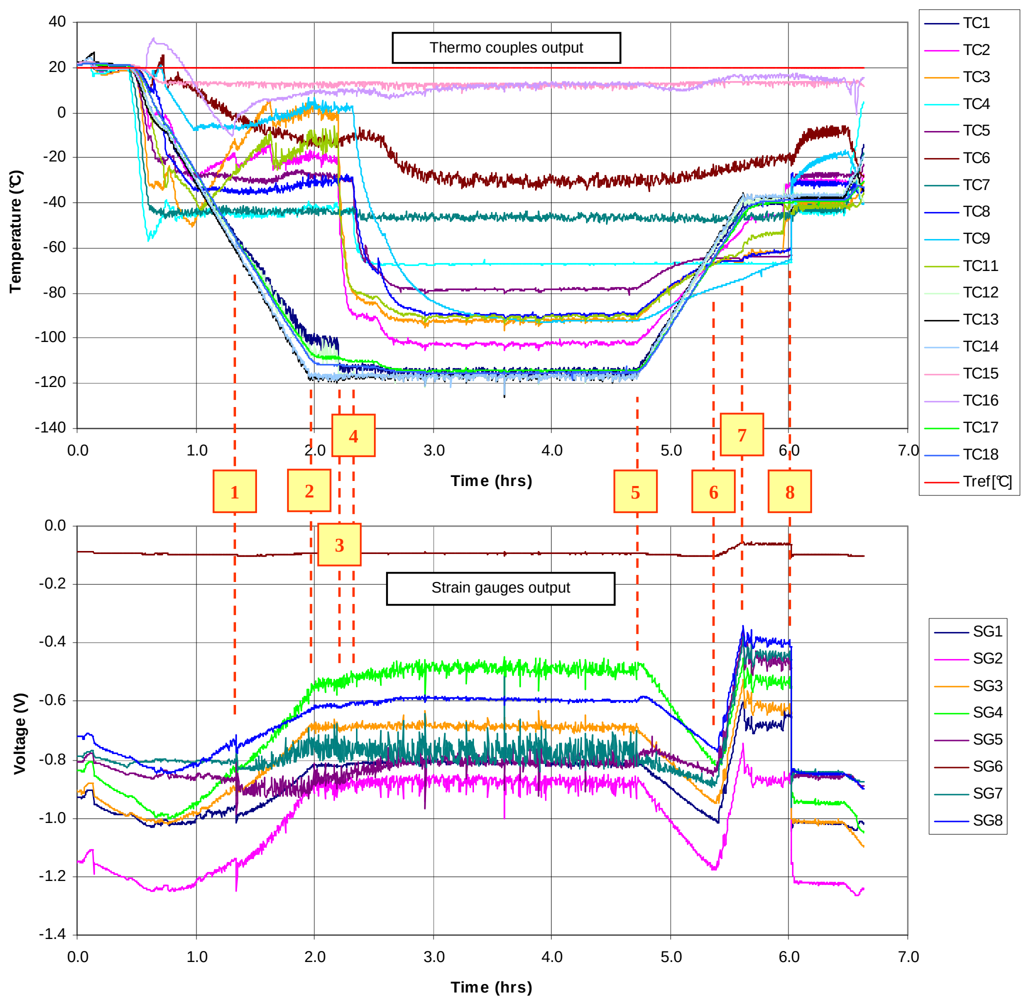


Figure 6-2: Measurements plot over a full test cycle period

Figure 6-2 shows a plot of measurements made with all thermo couples and strain gauges a full test cycle. The sequence of events is given below and indicated with numbers in Figure 6-2:

1. CO_2 in the condenser freezes solid
2. Freezing target temperature is reached
3. Zone heaters 1a and 1b are switched off
4. Zone heaters 2a and 2b are switched off
5. Thawing process commences
6. CO_2 in condenser reaches melting temperature
7. Thawing target temperature is reached
8. Feed lines complete thawing, allowing condenser CO_2 to expand

In Figure 6-2 it can be seen that the amount of noise on the measurements is larger for lower temperatures, as was also measured during calibration (Figure 4-2). The strain gauges appear to be very sensitive to temperature and temperature rate-of-change.
In principle this would have a negative effect on the accuracy of the measurements unless a reference pressure could be measured right before or after the MDP measurement. A pressure reference measurement just before the MDP determination is not possible. Fortunately due to the physics of the thawing sequence a reference measurement just after the MDP determination is possible. In Figure 6-2 this is can be done at point 8.
At that point the feed lines finally thaw and a sudden pressure drop occurs in the condenser. Throughout the system the CO_2 settles at the saturation pressure set by accumulator. In this particular case the accumulator had a final temperature of 13°C which corresponds to a saturation pressure of about 47Bar.

6.3 Maximum Design Pressure (MDP) results

In order to investigate of the MDP increase with temperature the following test sequence was followed:

- MDP determination at – 50 °C
- MDP determination at – 40 °C (repeated)
- MDP determination at – 30 °C
- MDP determination at – 20 °C
- MDP determination at – 40 °C (after a period at -120 C, repeated)

As the expected value of the MDP at -10 °C would mean that the material would enter the plastic deformation zone, no measurements at higher temperatures than -20 °C were performed.

To avoid a possible burst of the test tube early in the test campaign, the first four (4) tests were performed with a minimum temperature not far left from point C in Figure 6-1. This was done as before testing it was unknown whether solid expansion of carbon dioxide ice from B to C would cause tube damage. It was however found that this was not the case.

The test results at the several temperatures are shown in Figure 6-3 to Figure 6-6.

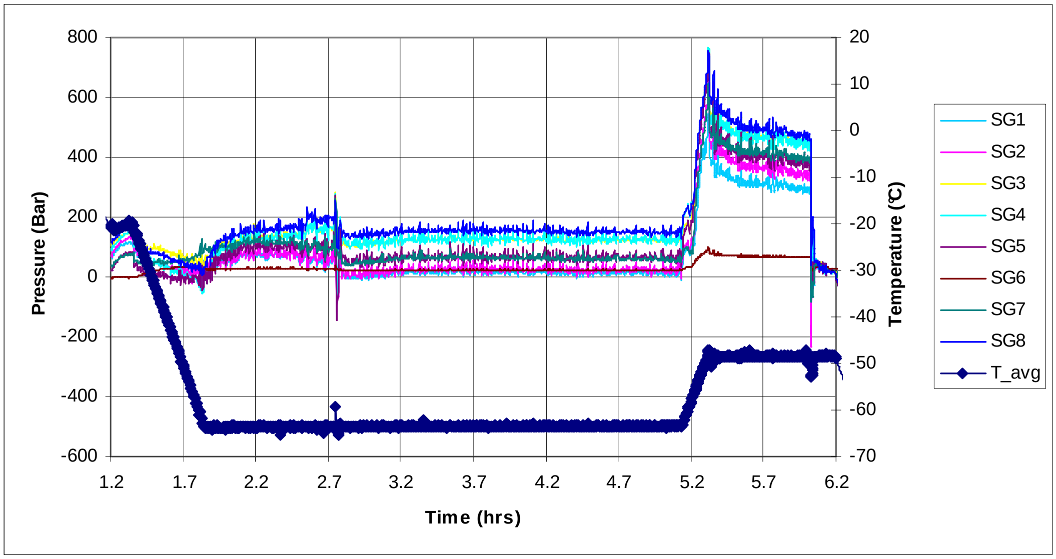


Figure 6-3: MDP determination at – 50 °C

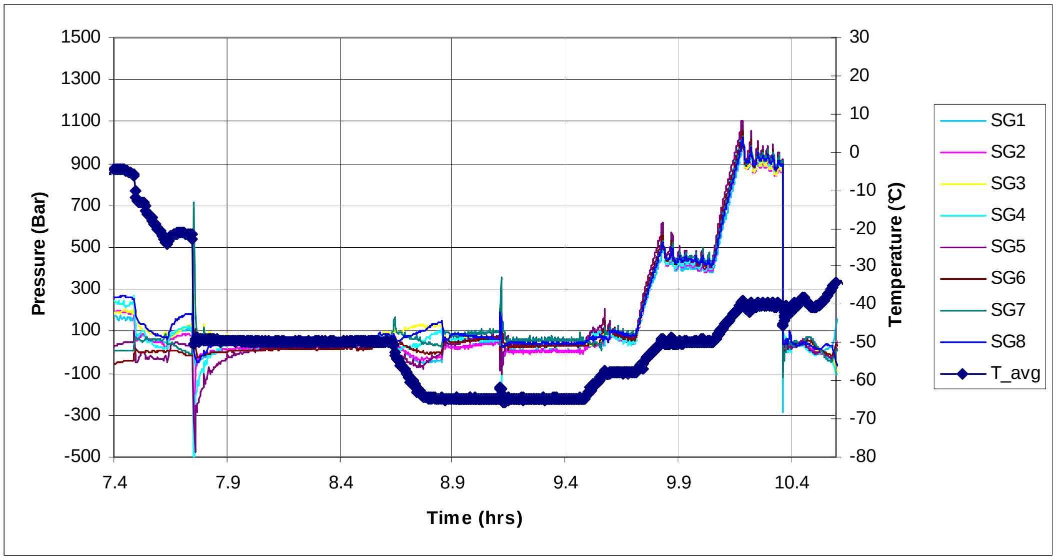


Figure 6-4: MDP determination at – 40°C

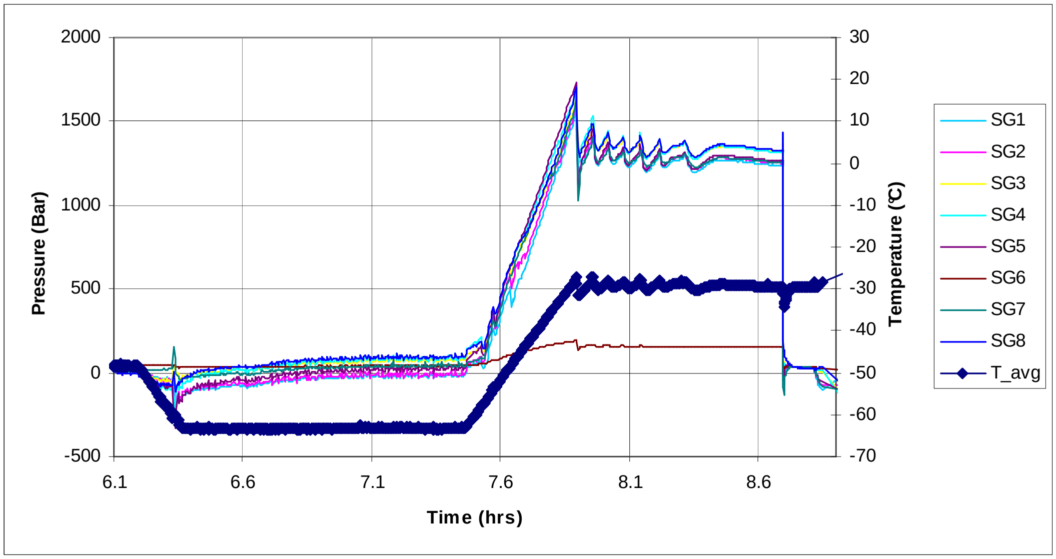


Figure 6-5: MDP determination at – 30°C

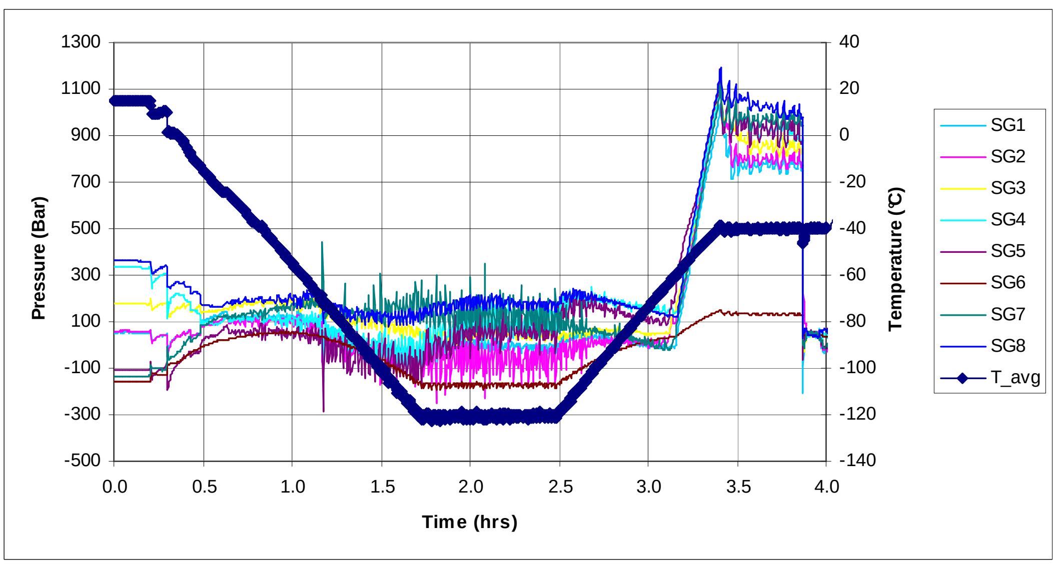


Figure 6-6: MDP determination at – 40°C after being cooled down to -120°C

Summarizing, the measured maximum pressures are plotted along the CO_2 melt line in Figure 6-7.

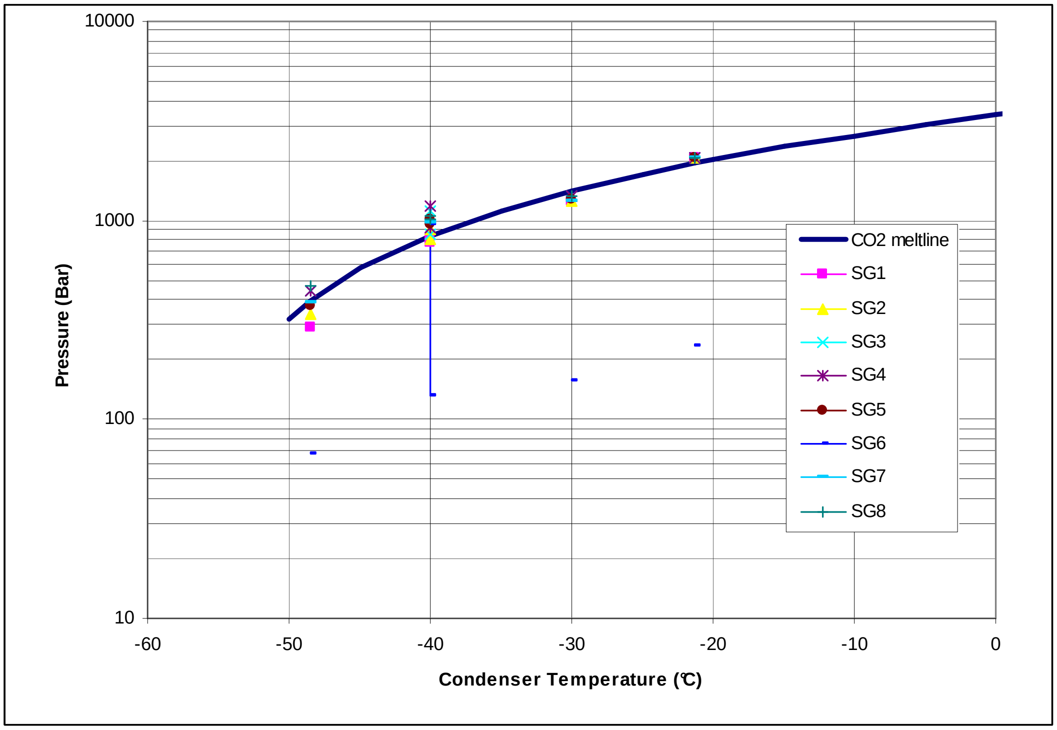


Figure 6-7: Maximum design pressure plotted along CO_2 melt line

7 Calculations based on test results

As the pressure build-up during thawing follows exactly the melt line, the Maximum Design Pressure of the condenser therefore is completely determined by the maximum radiator temperature of the unloaded tracker radiators after power down. This maximum temperature is calculated to be -5 °C as is shown in below figure.

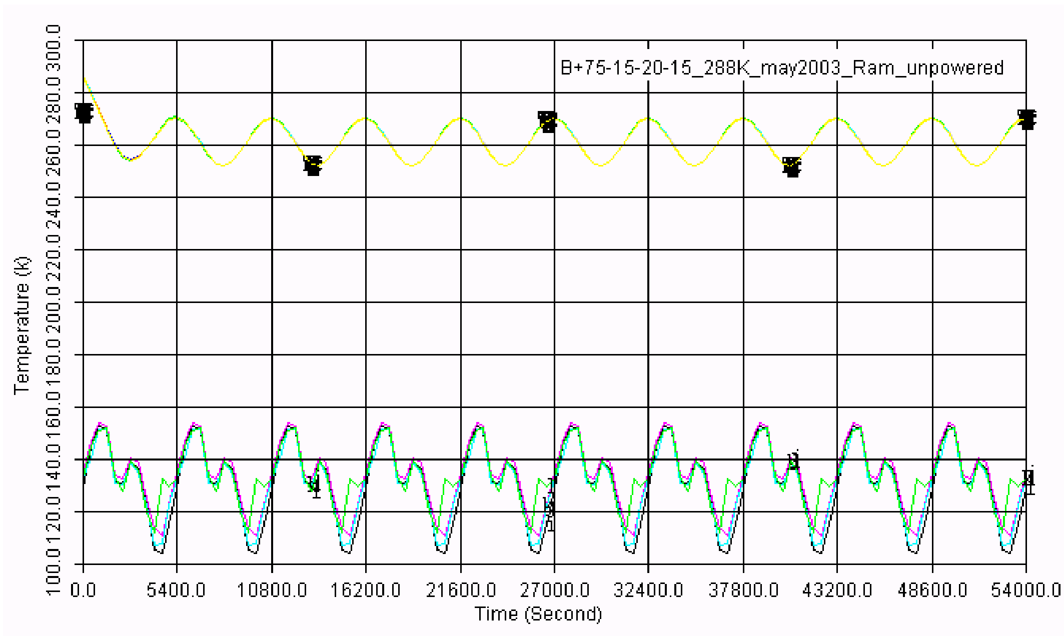


Figure 7-1: Hottest Case unpowered Ram-radiator temperature (Rad7hp_B+75-15-20-15_288K_may2003_unpower)

This maximum temperature corresponds to a pressure of approximately 3000 bar (melting line). Based on this pressure the material and tube thickness can be determined.

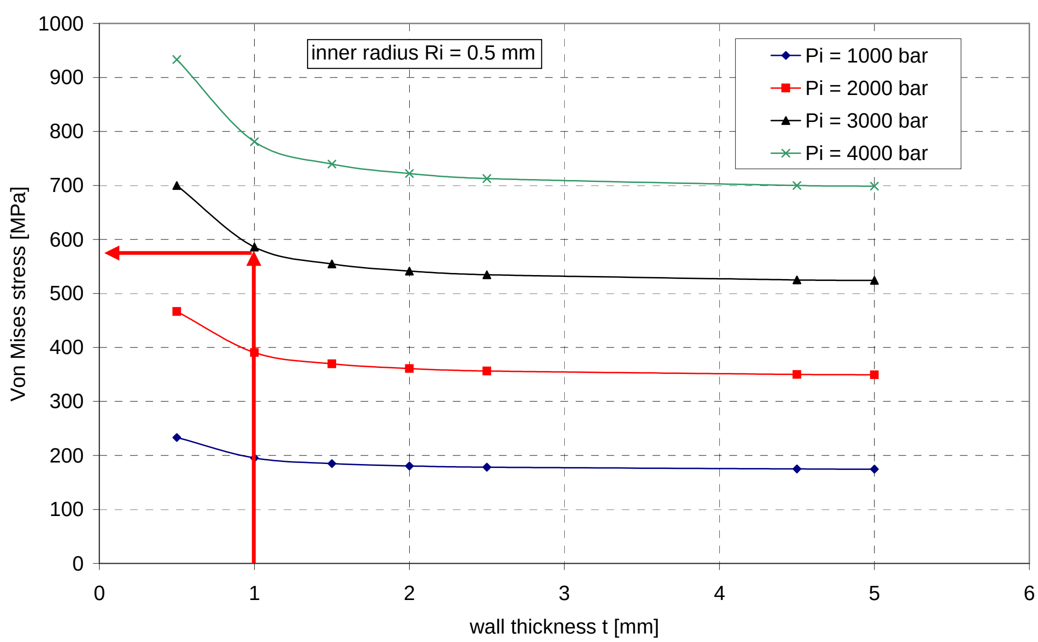


Figure 7-2: Von Mises stress as function of the wall thickness for different values of the inner pressure for an inner radius of 0.5 mm (from TN AMSTR-NLR-TN-022 issue 2.0).

In Figure 7-2 it can be found that a tube with r_i of 0.5 mm and a wall thickness of 1mm and an internal pressure of 3000 bar needs to sustain a von Mises stress of 586 MPa. Using a safety factor of 1.5 for yield [Ref 7, p39], [Ref 8, p119], the required material yield stress must be higher than 1.5x586=879 MPa. This is feasible for Inconel 718, see Table 7-1. An Inconel 718 tube with above dimensions will burst at 15638 bar [Ref. 4], [Appendix A expression (11)]; using a safety factor of 4.0 [Ref. 7, p39], [Ref. 8, p119], 15638/4.0=3910 bar is allowed. As a maximum of 3000 bar will occur while thawing, the envisaged tube easily complies the requirement. The material 'Inconel 718' was chosen as a likely candidate as it was used for similar reasons as part of the radiator of the International Space Station [Ref. 5].

Table 7-1: Material properties of Inconel 718

Density	8190 kg/m^3
Modulus of elasticity	200 Gpa
Yield stress	1034 MPa [Ref. 5]
Ultimate tensile stress	1280 MPa [Ref. 5]
Coefficient of thermal expansion	13.0 μm/m ^{o}C [Ref. 1]
Thermal conductivity	11.4 W/m ^{o}C [Ref. 1]
Specific heat	435 J/kg ^{o}C [Ref. 1]

8 Conclusions

- The MDP measurement with strain gauges showed to be successful and can be used to support freezing issues for other fluids.
- Negligible pressure rise is measured during the temperature rise from the -120 °C to -55 °C. This shows that no large expansion from the solid CO_2 is present in the condenser tube and indicates that during the solidification process the entrance and exits are blocked early in the freezing process so no major additional liquid CO_2 enters the condenser.
- The pressure build-up during thawing follows exactly the CO_2 melt line.
- The Maximum Design Pressure of the condenser can be taken from the CO_2 meltline once the maximum radiator temperature of the unloaded tracker radiators after power down is known.
- The condenser MDP is 3000 bar given that the maximum unloaded radiator temperature is -5 °C.
- A condenser comprising small diameter Inconel 718 tubing (d_{in} = 1.0mm, d_{out} = 3.0mm) is shown to withstand this pressure using a safety factor of 1.5 for yield and 4.0 for burst.

9 References

[1] Bibus Metals: Datasheet Inconel Alloy 718. W.Nr. 2.4668.

[2] Allvac: Technical data sheet Allvac 718 Alloy

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[5] Robert L. Broeren/R. John Duschatko, International Space Station Alpha Design-To-Freeze Radiators (SAE technical paper series 951652)

[6] Lit. S. 730 21125 Reine Stoffen in kondensierten Phasen. - Volumenanderung beim Schmelzen.

[7] MIL-STD-1522A 1984 Standard General Requirements for Safe design and Operation of Pressurized Missile and Space Systems

[8] MIL-HDBK-340 1985 Application guidelines for MIL-STD-1540B; test requirements for space vehicles

Appendix A: Stress equations

Circumferential:

$$\sigma_t = p_i\left(\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2}\right) \quad (1)$$

Radial:

$$\sigma_r = -p_i \quad (2)$$

Longitudinal:

$$\sigma_l = p_i\left(\frac{r_i^2}{r_o^2 - r_i^2}\right) \quad (3)$$

The longitudinal stress is zero if the ends of the tube are open.
As a measure of the limiting stress, the Von Mises stress is taken. It is defined as (for $\sigma_l = 0$):

$$\sigma_{VM} = \sqrt{\sigma_r^2 + \sigma_t^2 - \sigma_r\sigma_t} \quad (4)$$

Substituting the expressions for σ_r and σ_t yields:

$$\sigma_{VM} = p_i\sqrt{\alpha^2 + \alpha + 1} \quad (5)$$

Where:

$$\alpha = \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \quad (6)$$

Note that the relation between radii and wall thickness is given by:

$$r_o = r_i + t \quad (7)$$

Using a safety factor of 4.0 on the ultimate strength (which is the limiting value), the maximum allowable Von Misses stress becomes:

$$\sigma_{VM} = 0.25\sigma_u \quad (8)$$

Appendix A: Stress equations, continued

The expression for the bursting pressure, p_u , is a function of the ultimate tensile strength and the radii of the tube:

$$p_u = 2\sigma_u \frac{r_o - r_i}{r_o + r_i} \qquad (9)$$

commonly known as the mean diameter formula, is essentially empirical but agrees reasonably well with experiments for both thin and thick cylindrical tubes. For very thick tubes the formula:

$$p_u = \sigma_u \ln\frac{r_o}{r_i} \qquad (10)$$

is preferable. Greater accuracy can be obtained by using with this formula a multiplying factor that takes into account the strain hardening properties of the material:

$$p_u = \frac{2\sigma_y}{\sqrt{3}}(2 - \frac{\sigma_y}{\sigma_u})\ln\frac{r_o}{r_i} \qquad (11)$$